International Journal of Innovations in Science & Technology

P ISSN : 2618-1630
E ISSN : 2909-6130

Multirate Adaptive Equalization

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Citation | Anjum. M.Y.S, Anjum. M.A.R, Riaz. U, "Multirate adaptive Equalization", International Journal of Innovations in Science and Technology, Vol 3, Special Issue, pp: 119-125, 2021.
Received | Dec 13, 2021; Revised | Dec 18, 2021 Accepted | Dec 19, 2021; Published |Jan 1, 2022.

Finite Impulse Response (FIR) filter model emulates the Inter Symbol Interference (ISI) in a wireless communication channel. An equalizer, typically an Infinite Impulse Response (IIR) filter, behaves as an inverse filter to the FIR filter to remove the effects of the ISI. IIR filters are generally avoided due to tractability issues, and an FIR filter, with an adaptive signal processing algorithm to minimize the error due to the ISI, is deployed at the receiver. However, the filter is observed to quickly reach a steady state where further iterations do not yield a reduction in the error. This can be attributed to relatively slow variations in the steady state error which prevent further reduction of the errors. This work focuses on converting the low frequency error variations to high frequency variations by the use of multirate signal processing. As such, the steady state error can be damped as well, providing further reduction in the error and an enhanced adaptive filter performance.
Keywords: adaptive; filter; ISI; multirate; equalization.

INTRODUCTION

It has been well established that in order to remove the effects of ISI, an equalizer has to be employed at the wireless communication receiver [1]. Two choices are available in this regard: an FIR filter and an IIR filter. FIR filters are preferred over IIR filters due to their simplicity and ease of implementation. However, FIR filters are unable to sufficiently minimize the error due to ISI, and a certain allowance has to be made for the magnitude of error in order to deploy and reap the benefits of FIR filters at the wireless communication receiver.

A well-known criterion for making such an allowance is the Minimum Mean Square Error (MMSE) criteria [2]. Many adaptive signal processing algorithms are available that try to reduce the error based on the MMSE criteria. Least Mean Square Algorithm (LMS), Normalized LMS algorithm (NLMS), and Recursive Least Squares algorithm (RLS) are popular in this regard [3,11]. These algorithms are distinguished by their implementation

complexity and convergence speed, but an exclusive focus on the error analysis is not always to be found.

In iterative solvers, the error is observed to settle into a steady state after rapid initial convergence, where further iterations do not yield significant reduction in MSE [12]. This can be attributed to the ill-conditioning of the input covariance matrix, arising due to the disparity in the magnitude of its eigenvalues. Error associated with small magnitude eigenvalues dampens quickly whereas the error associated with large magnitude eigenvalues tends to linger on. Further observed is the frequency of the eigenvectors associated with large magnitude eigenvalues, which is relatively lower compared to the frequency of eigenvectors associated with small magnitude eigenvalues. These two factors, i.e., relatively large magnitude of an eigenvalue and the low frequency of its associated eigenvector, causes the MSE to settle into the steady state early, where in further reduction in MSE with increased number of iterations is not possible.

In this work, we proposed that the magnitude of larger eigenvalues can be reduced and the low frequencies of their associated eigenvectors can be converted to high frequencies by down sampling the error vector, wgicg will cause the error associated with large magnitude eigenvalues to dampen quickly as well. In this way, MSE can be reduced further and the filter convergence can be enhanced. This is the premise behind the presented work.

EQUALIZATION IN WIRELESS COMMUNICATION SYSTEMS

The equalization problem. Basic model of the equalization problem in a wireless communication system is depicted in Figure 1 [2].

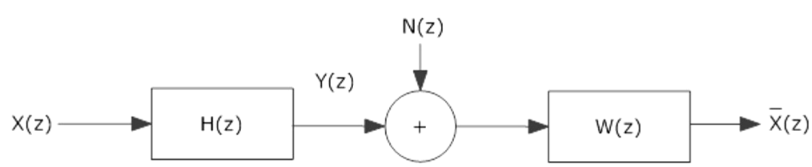

Figure 1. Model of equalization problem.

Output $Y\,(z)$ of a multipath channel $H(z)$ in the presence of Additive White Gaussian Noise (AWGN) $N(z)$ of zero mean and σ^2 variance can be expressed as:

$$Y\,(z) \;=\; H(z)X(z) \;+\; N(z) \tag{1}$$

With the corresponding MMSE equalizer $W\,(z)$ output being:

$$W(z) = \frac{\phi_{xx}(z)H(z^{-1})}{\phi_{xx}(z)\phi_{hh}(z) + \phi_{nn}(z)} \tag{2}$$

Such that$\phi_{xx}(z)$, $\phi_{hh}(z)$, and $\phi_{nn}(z)$ represent the power spectral densities of the input, channel, and noise respectively. MMSE equalizer endeavors retrieve $X(z)$ by minimizing the error between $X(z)$ and the equalizer output$\bar{X}(z)$.

Adaptive solution to equalization problem. Adaptive solution to the equalization problem is depicted in Figure 2.

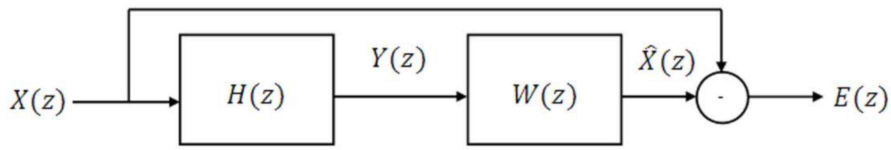

Figure 2. Modified equalization model.

An adaptive Wiener filter provides the ratio of the cross power spectral density $\phi_{xy}(z)$ to the auto power spectral density $\phi_{yy}(z)$ [2]:

$$W(z) = \frac{\phi_{xy}(z)}{\phi_{yy}(z)} \tag{3}$$

Which, in time domain, can be written in the following matrix form:

$$\mathbf{w} = \mathbf{R}^{-1}\mathbf{p} \tag{4}$$

(4) Is known as the Wiener-Hopf equation [2]. $\mathbf{w}$ Is the so-called Wiener filter and represents the desired response of the adaptive equalizer. $\mathbf{R} = E\{\mathbf{yy^H}\}$ Is the filer input autocorrelation matrix, and $\mathbf{p} = E\{\mathbf{xy^H}\}$ the input-output cross correlation vector.

ANALYSIS OF ERROR

Error controlling matrix. (4) Can be rewritten as:

$$\mathbf{Mw} = \mathbf{p} - \mathbf{Rw} + \mathbf{Mw} \tag{5}$$

Where $\mathbf{M}$ represents the error controlling matrix. (5) can be rearranged as:

$$\mathbf{w} = (\mathbf{I} - \mathbf{M}^{-1}\mathbf{R})\mathbf{w} + \mathbf{M}^{-1}\mathbf{p} \tag{6}$$

$\mathbf{I}$ Represents the identity matrix. (6) can be iteratively solved as [2]:

$$\mathbf{w[n+1]} = (\mathbf{I} - \mathbf{M}^{-1}\mathbf{R})\mathbf{w[n]} + \mathbf{M}^{-1}\mathbf{p} \tag{7}$$

Or:

$$\mathbf{e[n+1]} = (\mathbf{I} - \mathbf{M}^{-1}\mathbf{R})\mathbf{e[n]} \tag{8}$$

Such that $\mathbf{e[n]} = \mathbf{w[n]} - \mathbf{w}$, with $\mathbf{w}$ representing the exact solution to (4). (8) Shows that the convergence of error depends on the eigenvalues of the error controlling matrix $(\mathbf{I} - \mathbf{M}^{-1}\mathbf{R})$.

Computation of eigenvalues. Given that the equalizer $W(z)$ employed in Figure 2 is a two-tap filter, with the correlation factorα, then $\mathbf{R}$ in (4) can be represented as [2]:

$$\begin{bmatrix} 1+\alpha^2 & -\alpha & 0 & 0 \\ -\alpha & 1+\alpha^2 & -\alpha & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & -\alpha & 1+\alpha^2 \end{bmatrix} \tag{9}$$

With$\alpha = 1$, $\mathbf{R}$ can be viewed a second difference matrix with Dirichlet boundary conditions [12]. It has eigenvalues of the form:

$$\lambda^R{}_j = 2 - 2\cos\left(\frac{j\pi}{N+1}\right) \tag{10}$$

Where$j = 1,\ldots,N$, such that N represents the size and $\lambda^R{}_j$the $j-$the eigenvalue of the $\mathbf{R}$ matrix. Setting $\mathbf{M} = \mathbf{I}$ in the error controlling matrix $\mathbf{D} = \mathbf{I} - \mathbf{M}^{-1}\mathbf{R}$ leads to the following expression for the eigenvalues:

$$\lambda^D{}_j = 2\cos\left(\frac{j\pi}{N+1}\right) - 1 \tag{11}$$

Maximum value of $\lambda^{D}{}_{j}$ will be one when the cosine is zero, which is not satisfactory. In order for the error to converge, all the eigenvalues must be less than one. Selecting $\mathbf{M}^{-1} = \mathbf{I}/2$ causes all the eigenvalues of $\mathbf{D}$ to be less than one.

$$\lambda^{D}{}_{j} = \cos\left(\frac{j\pi}{N+1}\right) - 1 \tag{12}$$

First four eigenvalues of $\mathbf{D}$ computed from (12) are: $\cos(\pi/5) = 0.8090, \cos(2\pi/5) = 0.3090, \cos(3\pi/5) = -\cos(2\pi/5) = 0.3090, \cos(4\pi/5) = -\cos(\pi/5) = 0.8090$. Note that the lower frequency $\cos(\pi/5)$ has larger eigenvalue magnitude (0.8090) compared to the eigenvalue magnitude (0.3090) associated with the higher frequency$cos(2\pi/5)$. This will cause the higher frequency to dampen faster in iteration process. The lower frequency will dampen slowly and tend to linger on. If lower frequency is converted to a higher frequency, by the process of down sampling, it can be made to dampen faster like the higher frequency, and the convergence and the accuracy can be improved. The following general formula was derived for the eigenvalues of the error controlling matrix with an arbitrary value α:

$$\lambda^{D}{}_{j} = \alpha\cos\left(\frac{j\pi}{N+1}\right) - \frac{(\alpha^2 - 1)}{2} \tag{13}$$

An eigen distribution of $\mathbf{D}$ computed from (15) for $N = 10$ versus α is displayed in Figure 3 for illustration.

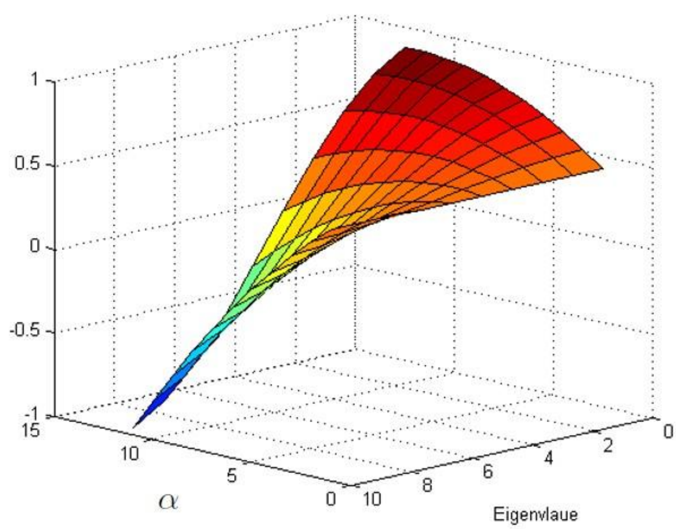

Figure 3. Eigen distribution of the error controlling matrix $\mathbf{D}$ for$N = 10$.

Computation of eigenvectors. The following general expression was derived for the eigenvectors of error controlling matrix $\mathbf{D}$ with an arbitrary valueα:

$$v_j[n] = \sin\frac{j\pi}{N+1}n \tag{14}$$

Eigenvector plot of $\mathbf{D}$ $(N = 10)$ versus different values of *a* is displayed in Figure 4. Note that the large eigenvalues are associated with low frequency eigenvectors, and vice versa.

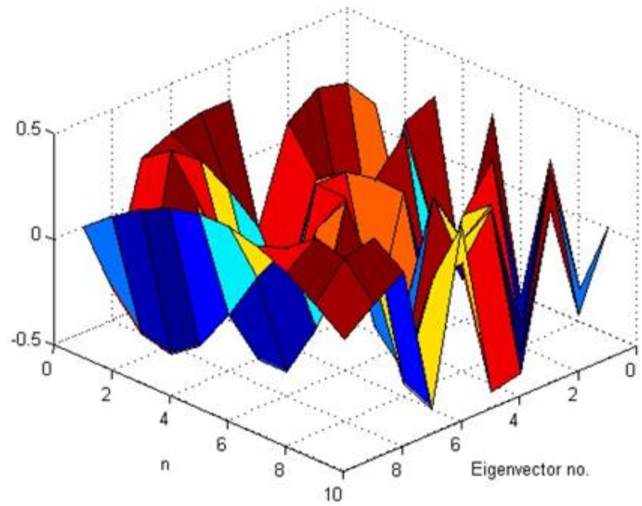

Figure 4. Eigenvectors of error controlling matrix **D** with $N = 10$ versus different values ofα.

DOWNSAMPLING AND ANALYSIS OF DOWNSAMPLED ERROR

Downsampling of eigenvalues. Now the error in (8) is downsampled by a factor of two, and the downsampled eigenvalues are compared with those of the original system$(N = 10, \alpha = 0.5)$. Downsampling is performed according to the following expression:

$$\lambda^{D}{}_{j} = \alpha\cos\left(\frac{j\pi}{N/2+1}\right) - \frac{(\alpha^2-1)}{2} \tag{15}$$

Downsampling will reduce the eigenvalues to five in contrast to the original ten. A graphical comparison of the original and the down sampled eigenvalues is displayed in Figure 5, which shows that the magnitude of larger eigenvalues is considerably reduced after down sampling.

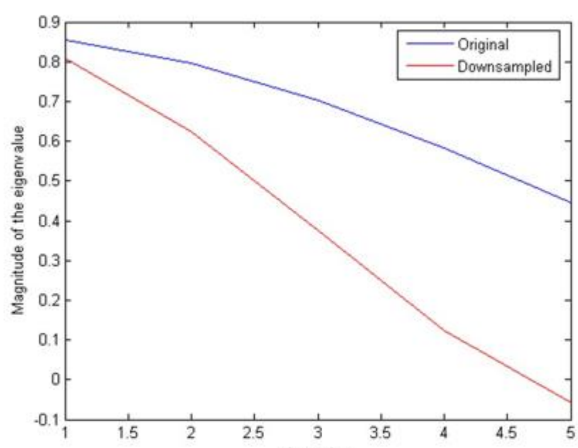

Figure 5. Graphical comparison of the downsampled and original eigenvalues of the error controlling matrix for$N = 10, \alpha = 0.5$, and downsampling factor of2.

Analysis of downsampled error. Re-writing (8) as:

$$\boldsymbol{e}[\boldsymbol{n}+\mathbf{1}] = \boldsymbol{D}\boldsymbol{e}[\boldsymbol{n}] \quad (16)$$

With $\mathbf{D} = \mathbf{I} - \mathbf{M}^{-1}\mathbf{R}$. Alternatively for (16):

$$\boldsymbol{e}[\boldsymbol{n}] = \boldsymbol{D}^{k}\boldsymbol{e}[\mathbf{0}] \quad (17)$$

For $k = 0$:

$$\boldsymbol{e}[\mathbf{0}] = c_1\boldsymbol{v}_1 + c_2\boldsymbol{v}_2 + \cdots + c_n\boldsymbol{v}_n \quad (18)$$

Multiplying (18) by $\mathbf{M}$:

$$\mathbf{Me}[\mathbf{0}] = c_1\mathbf{Mv_1} + c_2\mathbf{Mv_2} + \cdots + c_n\mathbf{Mv_n} \quad (19)$$

Leads to:

$$\mathbf{Me}[\mathbf{0}] = c_1\lambda_1\mathbf{v_1} + c_2\lambda_2\mathbf{v_2} + \cdots + c_n\lambda_n\mathbf{v_n} \quad (20)$$

Finally:

$$\mathbf{M^k e}[\mathbf{0}] = c_1{\lambda^k}_1\mathbf{v_1} + c_2{\lambda^k}_2\mathbf{v_2} + \cdots + c_n{\lambda^k}_n\mathbf{v_n} \quad (21)$$

Equation (21) shows that, for the error to dampen quickly, magnitude of the eigenvalues should be as small as possible; ideally, they must be zero. However, due to the poor conditioning of the input covariance matrix, a disparity can be found in the magnitude of eigenvalues. Large magnitude eigenvalues cause the error to enter into a steady state and prevent further reduction of MSE. The steady state error can be reduced by downsampling the error vector, which makes the associated eigenvalues to dampen quickly as well when their higher powers are taken in to account according to (21). A plot of the MSE achieved from the original and the downsampled error sequence is displayed in Figure 6. The plot confirms that the downsampled error reduces much faster than the original one.

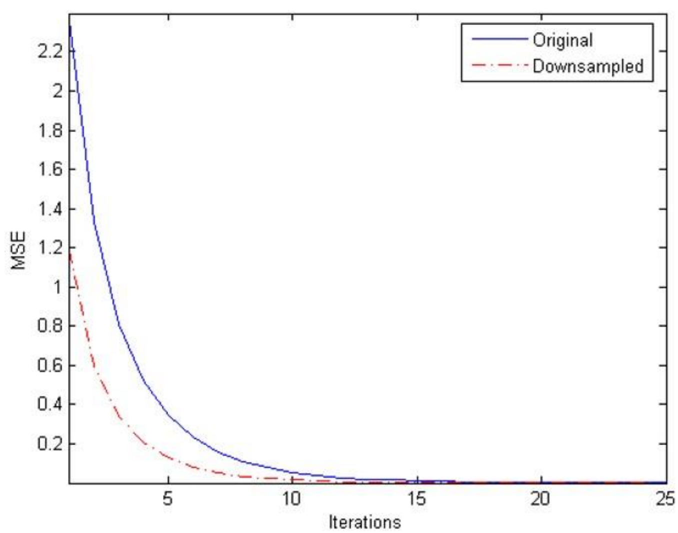

Figure 6. Comparison of the MSE achieved from the original and the downsampled error sequence.

CONCLUSION

An improvement in the error performance of adaptive wireless channel equalizer was found by downsampling the error vector by a factor of two. The improvement was demonstrated analytically and numerically. Further possibilities for improvement in error

performance of adaptive equalizer can be explored by increasing the downsampling factor beyond two.

Acknowledgement. No funding to declare.

Author's Contribution. Muhammad Yasir Siddique Anjum wrote the first draft of the manuscript, and conducted the data analysis. Muhammad Ali Raza Anjum provided technical expertise of multirate signal processing, and helped edit the manuscript. Usman Riaz supervised the study, provided factual review, and helped edit the manuscript.

Conflict of interest. There exists no conflict of interest for publishing this manuscript.

Project details. The research was not conducted as a part or result of a project.

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